Sparse Additive Generative Models of Text

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Generative models of text

Generative models are a powerful tool for understanding document collections.

- Classfication/clustering (Naive Bayes)
- Discover latent themes (LDA)
- Distinguish latent and observed factors (e.g. Topic-aspect models)

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Unifying idea: each class or latent theme is represented by a distribution over tokens, $P(w|y)$

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- Heuristic solutions like stopword pruning are hard to generalize to new domains.
- It would be better to focus computation on parameters that distinguish the classes.

Overparametrization

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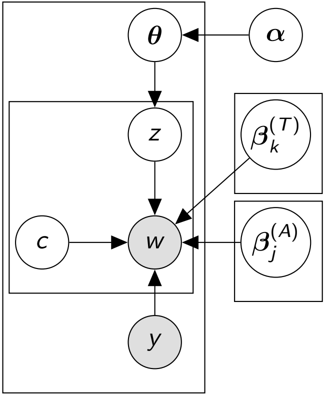
- An LDA **paper** shows 10 words per topic.
- What about the other $V - 10$ words per topic??
  - These parameters affect the assignment of documents...
  - But they may be unnoticed by the user.
  - And there may not be enough data to estimate them accurately.

Inference complexity

- Latent topics may be combined with additional facets, such as sentiment and author perspective.
- “Switching” variables decide if a word is drawn from a topic or from another facet.
- Twice as many latent variables per document!



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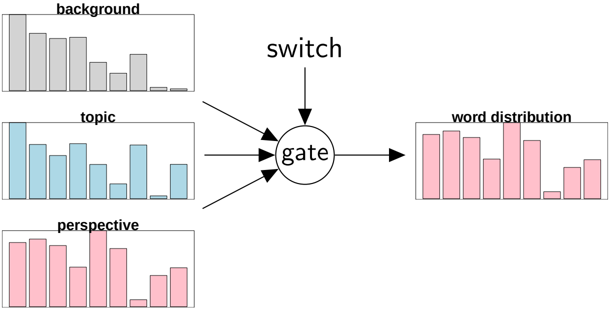
$$P(w|y, \mathbf{m}) \propto \exp\left(\mathbf{m} + \boldsymbol{\eta}_y\right)$$

  - $\mathbf{m}$ captures the background word log-probabilities
  - $\boldsymbol{\eta}$ contains sparse deviations for each topic or class
  - additional facets can be added in log-space

Sparse Additive Generative Models

A topic-perspective-background model using Dirichlet-multinomials:



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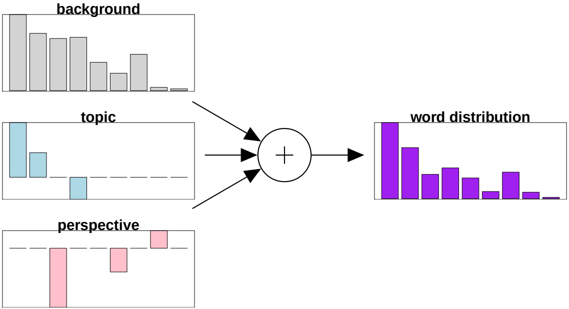

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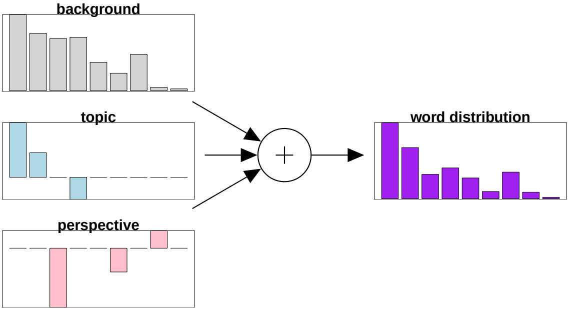

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Different notion of sparsity from sparseTM (Wang & Blei, 2009), which sets $Pr(w = i|y) = 0$ for many i.

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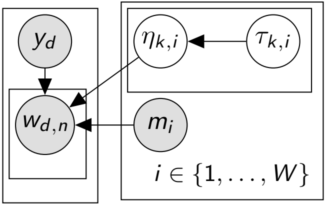
    - The variational distribution $Q(\boldsymbol{\tau})$
    - A **point estimate** of $\boldsymbol{\eta}$

Applications

- Document classification
- Topic models
- Multifaceted topic models

SAGE in document classification



- Each document d has a label y_d
- Each token $w_{d,n}$ is drawn from a multinomial distribution $\boldsymbol{\beta}$, where $\beta_i = \frac{\exp(\eta_{y_d,i} + m_i)}{\sum_j \exp(\eta_{y_d,j} + m_j)}$
- Each parameter $\eta_{k,i}$ is drawn from a distribution equal to $\mathcal{N}(0, \tau_{k,i})$, with $P(\tau_{k,i}) \sim 1/\tau_{k,i}$

Inference

- We maximize the variational bound

$$\ell = \sum_d \sum_n^{N_d} \log P(w_n^{(d)} | \mathbf{m}, \eta_{y_d}) + \sum_k \langle \log P(\boldsymbol{\eta}_k | \mathbf{0}, \boldsymbol{\tau}_k) \rangle + \sum_k \langle \log P(\boldsymbol{\tau}_k | \gamma) \rangle - \sum_k \langle \log Q(\boldsymbol{\tau}_k) \rangle ,$$

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- The gradient wrt $\boldsymbol{\eta}$ is,

$$\frac{\partial \ell}{\partial \boldsymbol{\eta}_k} = \mathbf{c}_k - C_k \boldsymbol{\beta}_k - \mathrm{diag}\left(\langle \boldsymbol{\tau}_k^{-1} \rangle\right) \boldsymbol{\eta}_k,$$

where

- $\mathbf{c}_k$ are the observed counts for class k
- $C_k = \sum_i c_{ki}$
- $\boldsymbol{\beta}_k \propto \exp(\boldsymbol{\eta}_k + \mathbf{m})$

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- We choose $Q(\tau_{k,i}) = \text{Gamma}(\tau_{k,i}; a_{k,i}, b_{k,i})$

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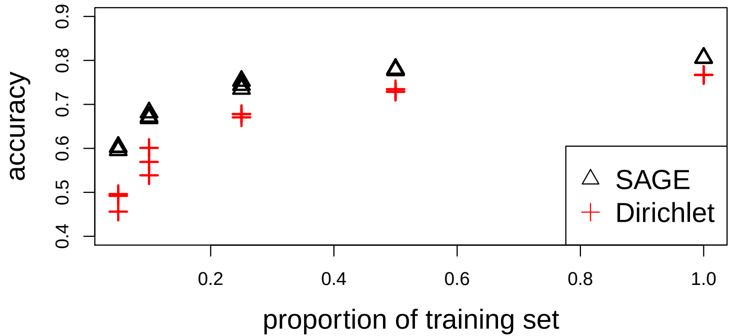
- We choose $Q(\tau_{k,i}) = \text{Gamma}(\tau_{k,i}; a_{k,i}, b_{k,i})$
- Iterate between a Newton update to a and a closed-form update to b

Document classification evaluation

- 20 newsgroups data: 11K training docs, 50K vocab



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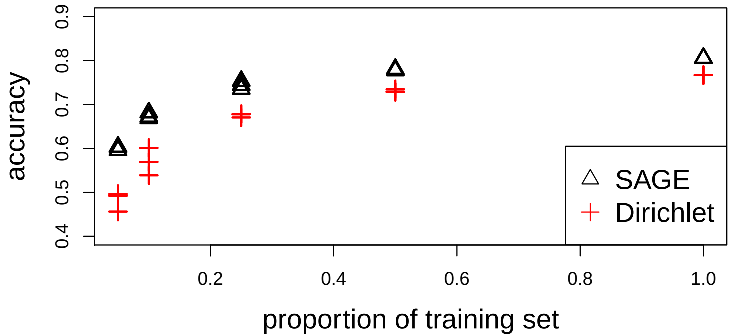
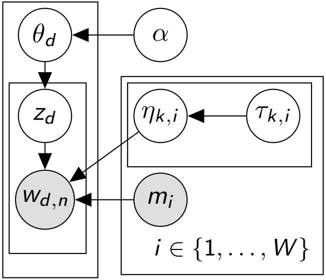


- Adaptive sparsity:
  - 10% non-zeros for full training set (11K docs)
  - 2% non-zeros for minimal training set (550 docs)

SAGE in latent variable models



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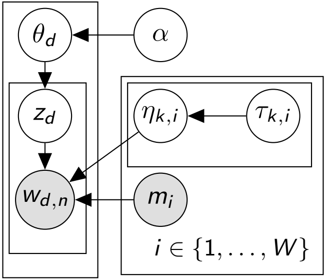


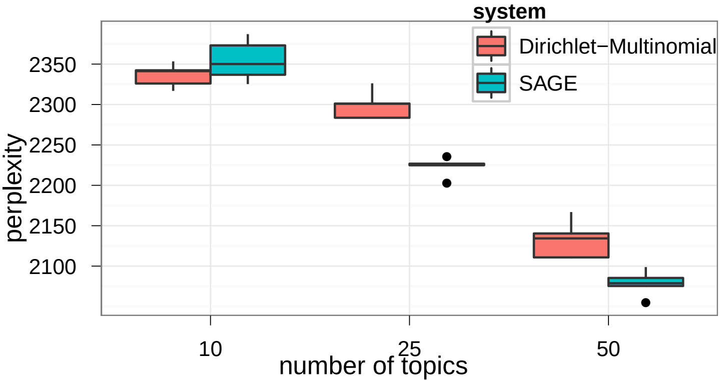
The gradient for $\boldsymbol{\eta}$ now includes **expected** counts:

$$\frac{\partial \ell}{\partial \boldsymbol{\eta}_k} = \langle \mathbf{c}_k \rangle - \langle C_k \rangle \boldsymbol{\beta}_k - \text{diag}\left(\langle \boldsymbol{\tau}_k^{-1} \rangle\right) \boldsymbol{\eta}_k,$$

where $\langle c_{ki} \rangle = \sum_n Q_{z_n}(k) \delta(w_n = i)$.

Sparse topic model results

- NIPS dataset: 1986 training docs, 10K vocabulary



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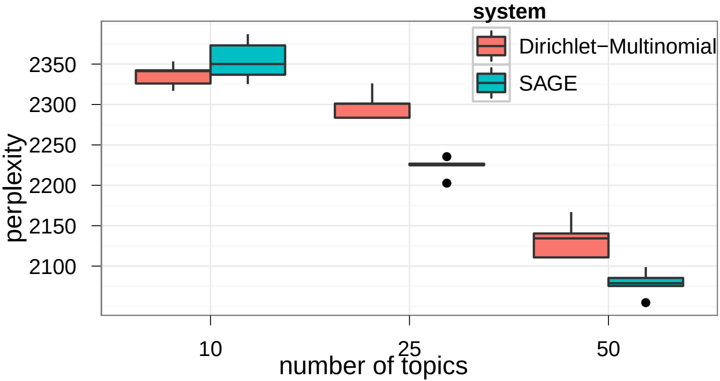
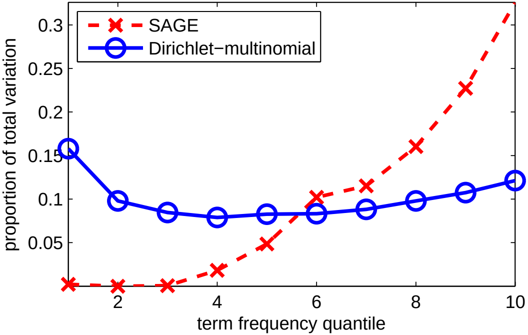


- Adaptive sparsity:
  - 5% non-zeros for 10 topics
  - 1% non-zeros for 50 topics

Sparse topic model analysis

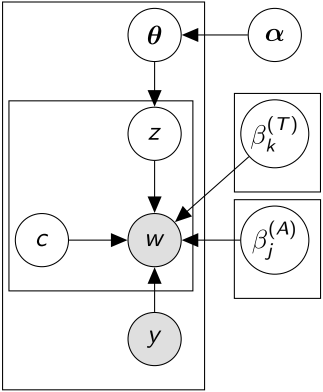
Total variation $= \sum_i |\beta_{k,i} - \overline{\beta}_i|$



Standard topic models assign the greatest amount of variation for the probabilities of the words with the least evidence!

Multifaceted generative models

- Combines latent topics $\beta^{(T)}$ with other facets $\beta^{(A)}$, e.g. ideology, dialect, sentiment



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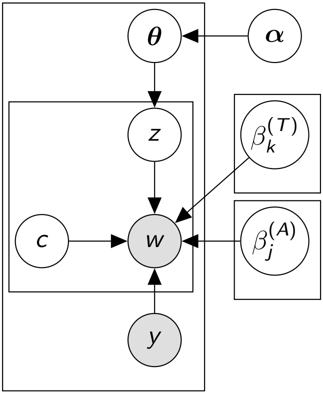

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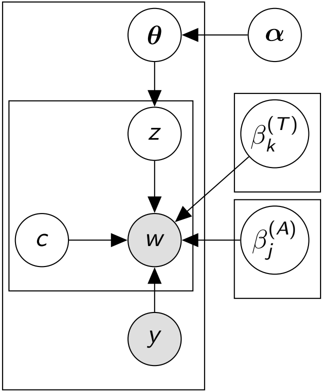
- There is one switching variable per token, complicating inference.



Multifaceted generative models in SAGE

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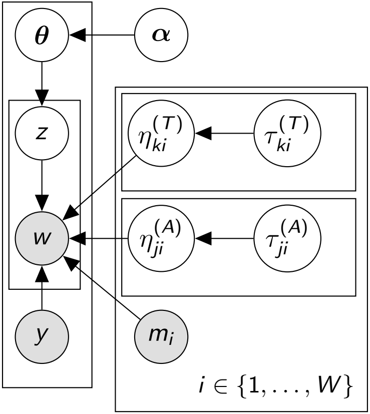

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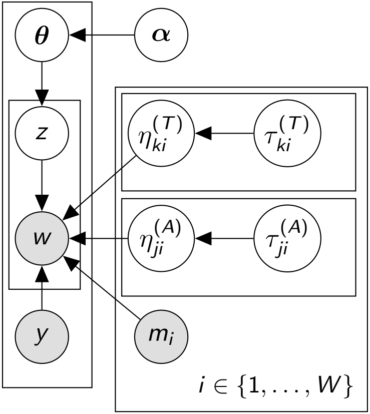

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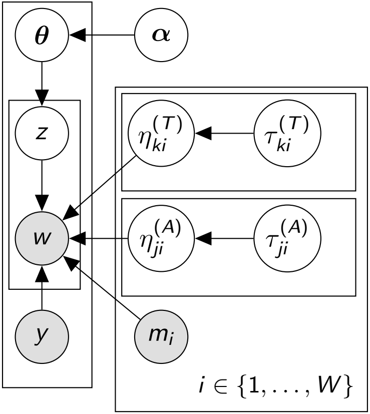

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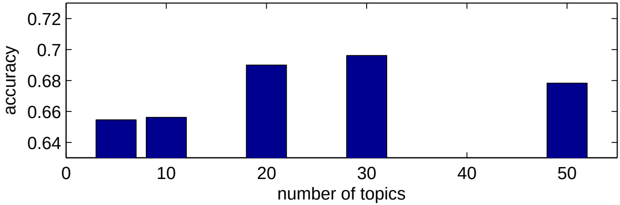

Results match previous best of 69% for Multiview LDA and support vector machine (Ahmed & Xing, 2010).

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- Thanks!

Example Topics

20 Newsgroups, Vocab=20000, K=25

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- wolverine punisher hulk mutants spiderman dy timucin bagged marvel
- gaza gazans glocks glock israeli revolver safeties kratz israel
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SAGE (Perplexity = 1090)

- ftp pub anonymous faq directory uk cypherpunks dcr loren
- disease msg patients candida dyer yeast vitamin infection syndrome
- car cars bike bikes miles tires odometer mavenry altcit
- jews israeli arab arabs israel objective morality baerga amehdi hossien
- god jesus christians bible faith atheism christ atheists christianity